Analysis of Some Combinatorial Properties of a 2D Torus

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Abstract

This paper introduces an exploration to the permuting ability of a 2D torus under deterministic XY routing. The research is carried out for a number of communication models, namely, unidirectional uniaxial, bidirectional uniaxial, unidirectional biaxial, and bidirectional biaxial. Necessary and sufficient conditions of blocking occurrence in a 2D torus for uniaxial models are expressed mathematically with the use of congruence notion from number theory. Examples of applying the technique to some permutations of either BPC (bit-permute-complement) or Omega class are given. Comparison of efficiency of different communication models is carried out. In particular, it is found that such important permutations as perfect shuffle and bit reversal are admissible to 2D torus under XY routing with unidirectional biaxial model.

Keywords: *2D torus, XY routing, permutations, blocking, congruence.*

Introduction

A variety of two-dimensional (2D) meshes were adopted in a number of actual multicomputer systems as interconnection networks. Meshes are particularly popular because of scalability, fault tolerance and simplicity. They are also suitable for VLSI implementation. However, a "classic" 2D mesh is not regular: a node degree depends on the localization of a node within the structure, so the corners have node degree 2, but not 4, and should be treated in a special way. A torus is defined as a mesh with wraparound connections which give it regularity and symmetry. The diameter of a torus is only half the diameter of a mesh and all nodes have the same number of neighbors. The 2D torus 4 x 4 is shown in Fig. 1.

The extensive literature on various aspects of packet routing in 2D meshes and tori was reviewed by Grammatikakis *et al*. (1998). Usually in monographs 2D connecting topologies are introduced together with the description of dimension-order or *XY* routing strategies. Such simple deterministic strategies, despite of being extremely vulnerable to faults and highly blocking, are very common.

However, information about their efficiency is relatively scanty, and publications on this subject are few in number. This paper is a revised and advanced version of an earlier paper by the author (Veselovsky 2006) and it is written in an attempt to contribute to the field applying both analytic and computational approaches for a variety of communication models but with the use of the same deterministic *XY* routing. So here some specific combinatorial properties of a 2D torus as a connecting topology are explored.

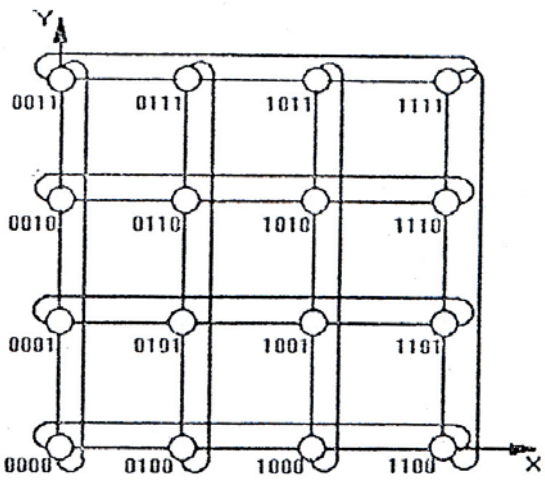

Fig. 1. The two-dimensional torus 4x4.