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A New Fuzzy Relational Product for Similarity Testing With an Application in Social Network Analysis

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Abstract

This paper describes a new fuzzy relational product that can be applied to two fuzzy relations to produce a new fuzzy relation, which contains the degrees of similarity between its elements. These degrees of similarity are determined according to relationships between the elements and their corresponding third-party elements that are defined in the two original fuzzy relations. This new fuzzy relational product has been developed as part of a research on social network analysis in an attempt to determine relationships among political figures based on their (current) opinions.

***Keywords**: Fuzzy relation, fuzzy relational product, social network analysis, political network, commendation relation, complaint relation.*

Introduction

Relationships among people in our society become more and more important in today's small world of information age. What people think and do affect others in the way never happened before. However, relationships among people are very complex, very difficult to understand, yet very dynamic. There is a definite need for good frameworks or tools that can be used to determine and analyze such relationships so that the society can be better prepared for the consequences that may be caused by the said relationships.

Wasserman and Faust (1994) discuss one field of study trying to systematically analyze the relationships among people in terms of Social Network Analysis in detail. In an attempt to enhance the social network analysis, fuzzy (binary) relations as discussed in Klir and Folger (1988) were used by the authors to develop an alternative framework for the analysis. In this alternative framework, certain fuzzy relational products that were discussed in Kohout and Bandler (1985; and 1990), as well as a new one proposed by the authors (Santiprabhob and Dowpiset 1999), were employed to determine relationships among a group of people in focus.

Political Network Environment

Since political figures are people who have a great deal of influence on our well-being, our research used political network as a model to develop a new framework for social network analysis. Due to the multi-party structure of Thai politics, politicians here change their opinions or even switch sides very dynamically. They sometimes behave so just simply to gain bargaining power within their coalition, either the government or the opposition. Such trends can be detected from what and how they discuss matters in public.

In our framework, news from an on-line source was used to create fuzzy relations that relate politicians to current topics of discussion. Two types of fuzzy relations were created, based on the feeling of each speaker towards the topics being discussed. In each fuzzy relation, there were *Actors* and *Topics*. The first fuzzy relation, called *Commendation* relation (Cm), related topics with actors who have positive feeling towards the topics being

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discussed. Such positive feeling was indicated in the news by keywords like *agree*, *support*, *praise*, etc. The other fuzzy relation, called *Complaint* relation (Cp), related topics with actors who have negative feeling towards the topics being discussed. In this case, the negative feeling was indicated in the news by keywords like *disagree*, *disbelieve*, *refuse*, etc. Fuzzy degrees for each pair of actor and topic were assigned manually according to the keywords found in the news. Where 0.0 means that the topic was not discussed by the actor, 0.5 means that the topic was discussed in a very neutral way by the actor, and 1.0 means that the actor had a very strong opinion about the topic (either to commend or to complain). Fuzzy degrees between 0.0 and 0.5 were not used, while fuzzy degrees between 0.5 and 1.0 were used to signify different levels of commendation or complaint. An example of such a fuzzy relation is shown in Fig.1.

	T_1	T_2	T_3	T_4	T_5	T_6	T_7
A_1	0.6	0.6	0.0	0.6	0.0	0.0	0.0
A_2	0.7	0.5	0.0	0.8	0.0	0.0	0.0
A_3	0.0	0.0	0.0	1.0	0.8	0.9	0.7
A_4	0.7	0.0	0.6	0.9	0.0	0.0	0.0
A_5	0.0	0.0	0.0	0.9	0.8	0.8	0.8
A_6	0.0	0.0	0.0	1.0	0.8	0.9	0.0

Fig. 1. Example of Commendation or Complaint fuzzy relation

Existing fuzzy relational products

Two of the existing fuzzy relational products, namely *Circle* product and *Square* product, which were discussed in Kohout and Bandler (1985;1990), were first employed by the authors to determine relationships between pairs of actors with respect to topics discussed by them.

Membership degrees of elements in a fuzzy relation resulted from an application of the *Circle* product on two fuzzy relations *R* and *S* can be defined as:

$$\mu_{R\circ S}(x_i, y_k) = \bigvee_j (\mu_R(x_i, z_j) \wedge \mu_S(z_j, y_k))$$

While membership degrees of elements in a fuzzy relation resulted from an application of the *Square* product on two fuzzy relations *R* and *S* can be defined as:

$$\mu_{R\square S}(x_i, y_k) = \bigwedge_j (\mu_R(x_i, z_j) \leftrightarrow \mu_S(z_j, y_k))$$

Where $\leftrightarrow$ is defined as

$$\mu_A(x) \leftrightarrow \mu_B(y) = \mu_A(x) \rightarrow \mu_B(y) \wedge \mu_A(x) \leftarrow \mu_B(y)$$

For the purpose of our research, the fuzzy implication operator $\rightarrow$ is defined using the *Standard Strict* operator as discussed in Bandler and Kohout (1980).

$$\mu_A(x) \rightarrow \mu_B(y) = 1 \text{ if } \mu_A(x) \le \mu_B(y)$$
$$\mu_B(y) \text{ if } \mu_A(x) > \mu_B(y)$$

Let's call the fuzzy relation in Fig.1, C. What we want to look at are the fuzzy relations that resulted from the applications of the above two fuzzy relational products on C and its transpose C^{-1}. Fig. 2 shows the result of fuzzy relation $C \circ C^{-1}$, and Fig. 3 of $C\square C^{-1}$.

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0.6	0.6	0.6	0.6	0.6	0.6
A_2	0.6	0.8	0.8	0.8	0.8	0.8
A_3	0.8	0.8	1.0	0.9	0.9	1.0
A_4	0.6	0.8	0.9	0.9	0.9	0.9
A_5	0.6	0.8	0.9	0.9	0.9	0.9
A_6	0.6	0.8	1.0	0.9	0.9	1.0

Fig. 2. Resulting fuzzy relation $C \circ C^{-1}$

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	1.0	0.5	0.0	0.0	0.0	0.0
A_2	0.5	1.0	0.0	0.0	0.0	0.0
A_3	0.0	0.0	1.0	0.0	0.7	0.0
A_4	0.0	0.0	0.0	1.0	0.0	0.0
A_5	0.0	0.0	0.7	0.0	1.0	0.0
A_6	0.0	0.0	0.0	0.0	0.0	1.0

Fig. 3. Resulting fuzzy relation $C\square C^{-1}$

With an application of the *Circle* product, the resulting fuzzy relation shows the relationships between pairs of actors with the fuzzy degrees specifying the degree to which actors in each pair discuss at least one common topic in the same feeling. On the other hand, the fuzzy relation resulted from an application of the *Square* product has the fuzzy degrees that specify the degree to which actors in each pair discuss exactly the same topics in the same feelings.

As can be seen from the result in Fig.2, with the *Circle* product, every actor seems to relate to every other actor. This is simply because every one in our particular case here had an opinion on topic T_4 to some degree. It can be concluded that, in general, the *Circle* product by itself does not help us very much in determining the relationships among the actors. The resulting relation does not tell us who is with whom in terms of their social networks/groups.

On the other hand, the result of the *Square* product, as shown in Fig.3, seems to yield some sort of grouping. In this example, two networks/groups were identifiable. The first network consists of A_1 and A_2. And, the second network seems to have A_3 and A_5 as its members. However, if we look more closely at the original fuzzy relation C in Fig.1, we would find that the resulting networks were correct but a little too stringent.

For example, A_6 has exactly the same opinion as A_3 in all the topics but one, namely topic T_7. What A_3 says, A_6 says with the same feelings, and what A_3 does not have an opinion on, A_6 also does not say anything, except for the topic T_7, on which A_3 has an opinion but A_6 does not have any. In reality, it is very likely that A_3 and A_6 belong to the same (political) network/group, but the *Square* product fails to identify the fact. This is due to the semantics of the *Square* product, which requires exact matches of opinions on all the topics discussed by the actors concerned.

New fuzzy relational product

To fill the gap between the results of the two existing fuzzy relational products discussed above, the authors have proposed a new fuzzy relational product called *Similarity* product. This new product looks at the difference in the opinions of two actors in each topic in order to find the degree of similarity, then average out such degrees of similarity over all the topics discussed. The said *Similarity* product is defined as

$$\mu_{R\otimes S}(x_i, y_k) = \frac{\sum\limits_{j} (1 - |\mu_R(x_i, z_j) - \mu_S(z_j, y_k)|)}{\Sigma j}$$

In the definition above, the absolute difference between two actors on the same topic is defined in the term

$$|\mu_R(x_i, z_j) - \mu_S(z_j, y_k)|$$

Therefore, the degree of similarity between two actors with respect to their opinions on a particular topic can be defined as

$$1 - |\mu_R(x_i, z_j) - \mu_S(z_j, y_k)|$$

The *Similarity* product, then, averages out the degrees of similarity over all the topics concerned.

Applying this new *Similarity* product to the original fuzzy relation C and its transpose yields the resulting fuzzy relation $C \diamond C^{-1}$ as shown in Fig.4.

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	1.0	0.9	0.4	0.8	0.4	0.5
A_2	0.9	1.0	0.5	0.8	0.5	0.6
A_3	0.4	0.5	1.0	0.5	1.0	0.9
A_4	0.8	0.8	0.5	1.0	0.5	0.6
A_5	0.4	0.5	1.0	0.5	1.0	0.9
A_6	0.5	0.6	0.9	0.6	0.9	1.0

Fig.4. Resulting fuzzy relation $C \diamond C^{-1}$

With its different semantics, the *Similarity* product can identify networks/groups of actors based on the overall similarities with respect to both the topics they discuss and the ones they do not discuss. The *Similarity* product does not discount the potential similarity of two actors simply because of a single discrepancy as in the case of the *Square* product.

From the resulting fuzzy relation in Fig.4, using α-cut of 0.6, we can identify two networks/groups. The first one consists of A_1, A_2 and A_4. While the second network consists of A_3, A_5 and this time also A_6. If we further look at the resulting fuzzy degrees of the relation in Fig. 4, we would find that they cover and agree with the results from an application of the *Square* product as shown in Fig.3. The pairs of actors that have been identified as strongly related by the degrees of "exact match" in Fig.3 also have very high degrees of "similarity" in Fig.4.

Political Networks Analysis

Based on our political network model, networks/groups of politicians can be analyzed by applying appropriate fuzzy relational products to the *Commendation* relation (Cm) and/or the *Complaint* relation (Cp). The *Square* product is used when strong/exact matches of actors are to be determined. On the other hand, the new *Similarity* product can be used when overall similarities among the actors are to be identified. According to the semantics of the *Commendation* relation, the *Complaint* relation, the *Square* product, and the *Similarity* product, the following results can be obtained.

- $Cm \square Cm^{-1}$ or $Cp \square Cp^{-1}$ yields degrees to which actors are exactly matched in terms of their opinions – actors who are definitely on the same sides.
- $Cm \diamond Cm^{-1}$ or $Cp \diamond Cp^{-1}$ yields degrees to which actors have similar opinions – actors who are likely on the same sides.
- $Cm \square Cp^{-1}$ yields degrees to which actors have exactly opposing opinions – actors who are definitely on the opposite sides.
- $Cm \diamond Cp^{-1}$ yields degrees to which actors have in overall opposing opinions – actors who are likely on the opposite sides.

Note that people in general, and politicians in particular, have complicated dynamic behavior and change their ideas from time to time. The analysis using fuzzy relations and fuzzy relational products discussed above must be done with respect to a certain time period. Besides the networks/groups of actors that we can identify, we are also able to detect interesting cases where the very same person (could be some notoriously inconsistent, no principle, no-nonsense politician!) contradicts him/herself. Such a case is revealed by a high fuzzy degree on the same person (at the diagonal) of the resulting fuzzy relation $Cm \square Cp^{-1}$ or $Cm \diamond Cp^{-1}$.

Note also that just like the *Square* product, the new *Similarity* product produces a very high degree of similarity, in fact a degree of 1.0, for any pair of actors who both discuss nothing at all. They are considered similar (or indeed the same) due to the fact that they have the same "no opinion." *Similarity* product also produces a high degree of similarity for any pair of actors who, most of the time, are silent, even though they may have some little differences in very few opinions they do express. For the purpose of analysis, if no opinion is regarded as a lack of information rather than the similarity between the two actors considered, the fuzzy relation resulted from an application of the *Circle* product should be used to intersect with the resulting fuzzy relation concerned. This would guarantee that there must be at least one common topic discussed by any pair of actors that have a degree of similarity greater than 0.0 in the final result.

Conclusion

In this paper, we have outlined a framework in which fuzzy relations and fuzzy relational operators are used to analyze relationships among people based on their opinions. Political network environment is used as a model to demonstrate the proposed framework. However, it can obviously be seen that the framework is general enough for application to a wide range of cases in social network analysis. In addition to the social network analysis framework proposed, a new fuzzy relational product called *Similarity* product is introduced to fill the gap between the *Circle* product and the *Square* product. Using this new *Similarity* product, the results of the analysis will better reflect the actual relationships that exist in reality among the people being considered.

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