

A Critical Study on the Use of the Gaussian Approximation in Optical Code-Division Multiple-Access Networks

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Abstract

The performance evaluation of optical sparse codes in code-division multiple-access (CDMA) networks is considered for the case of multi-access interference (MAI). A comparison with the standard Gaussian approximation (SGA) shows that the said approximation overestimates the performance of the codes and should not be used for small bit error rates (BERs) due to the weak convergence of the central limit theorem (CLT) for most cases of practical interest. It is also shown that the worst case BER is observed for chip-asynchronous optical CDMA networks.

Keywords: Code-division multiple-access networks, optical sparse codes, Gaussian approximation, central limit theorem, weak convergence, entropy, uniform distribution.

Introduction

The standard Gaussian approximation (SGA) based on the normal distribution and graphically represented by the bell curve is frequently used for the performance evaluation of *wireless* code-division multiple-access (CDMA) networks. The applicability of this approach is based on some assumptions concerning the probability density function (pdf) of the cross-correlations between different codewords for arbitrary delays. It is assumed that the *bipolar* pseudo-noise (PN) chip sequences to be used for direct sequence (DS) modulation of the information bits are being designed in such a way that the mean value of the said pdf equals zero. The maximum of the pdf is observed for cross-correlation equal to zero demonstrating the good quasi-orthogonal properties of the codewords, and also the pdf is almost symmetric around the mean. This will result in a faster convergence to the bell curve of the *sum distribution* of the normalized $(M-1)$ -fold *continuous* convolution of the pdf distribution due to multi-access interference (MAI) for M simultaneous users in the network. The term convergence is considered in the weak sense

that the bit error rates (BERs) obtained as a result of the integration of the tails of the sum pdf and the bell curve for a fixed decision threshold should be within the same order of magnitude and the central limit theorem (CLT) can be used in first approximation with the increase of M . Such a weak convergence can be observed for bit error rates greater than 0.001 when a significant part of the tail of the distribution contributes to the BER. For lower bit rates however, the convergence rapidly degrades and the exact normalized $(M-1)$ -fold convolution of the pdf distribution should be used for the calculation of the BER instead of SGA.

In the chip-synchronous case, the probabilities of the discrete distribution of the cross-correlations between different codewords for chip delays being divisible by the chip duration are taken into consideration. A central clock for chip synchronization of all simultaneous users is required in the network. A normalized $(M-1)$ -fold *discrete* convolution of the discrete distribution takes place for M simultaneous users. The BER is improved for the same decision threshold due to the reduced value of the standard deviation (SD) of the sum discrete distribution but the convergence to the