The Logistic Model of Growth

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Abstract

This article examines the popular logistic model of growth from three perspectives: its sensitivity to initial conditions; its relationship to analogous difference equation models; and the formulation of stochastic models with mean logistic growth. The results indicate that although the logistic model is appealing in terms of its simplicity, its appeal is questionable in terms of its realism.

Keywords: *Real-valued function, S-shaped data, difference equations, stochastic models.*

Introduction

It is common in many fields of study to model the values of a variable of interest $N(t)$ over time, t, using an autonomous differential equation of the form,

$$\frac{dN(t)}{dt} = F(N(t)), \qquad (1)$$

where $N(t)$ is a continuous real-valued function of time. In the case where realistically $N(t)$ is an integer-valued function of time, Eq. (1) is a model of the mean value of the integer-valued process. Assuming that the coefficient of variation of that process is small, the continuous model in Eq. (1) is used to represent $N(t)$. If $\frac{1}{N(t)}\frac{dN(t)}{dt}$ is constant, then the growth in $N(t)$ is said to be density independent and otherwise it is density dependent.

In many situations, experimental values of $N(t)$ exhibit an *S*-shaped graphical representation and although many possible functions $F(N(t))$ may be used in Eq. (1) to produce models with sigmoid growth curves, the Verhulst (1838) logistic model in Eq. (2) below is certainly one of the most popular (Hazen 1975),

$$\frac{dN(t)}{dt} = \lambda\,[K - N(t)]\,N(t), \qquad (2)$$

where usually $N(0) = N_0 < K$, $\lambda > 0$, K is referred to as the carrying capacity (i.e., an upper bound on the value of $N(t)$), and the product λK is the intrinsic rate of increase. The model in Eq. (2) has been studied and used extensively over a long period of time by researchers in demography, the biological sciences, ecology, genetics, applied statistics (logistic regression), software metrics, and many other fields of study (Feller 1940; Bertalanffy 1941; Lotka 1907, 1956; Keyfitz 1968; Pielou 1969; Hoppensteadt 1975; Turner *et al*. 1976; Walpole *et al*. 2002).

However, despite the popularity of the logistic model, which, as it will be seen, is probably based more on its simplicity than its realism, some of its features, and its relationship to analogous models using difference equations and stochastic growth models are often not well understood by researchers who use the model simply on the basis of the plausibility of its sigmoid growth curve. In particular, Feller (1940) warned against blind faith in the use of the logistic. He considered *S*-shaped data from an experiment and then selected several *S*-shaped functions at random. Applying the usual criteria for best fit, he ranked the various functions. The results were that the logistic fitted the data worse than the other selected functions. His conclusion was that the recorded agreement between the